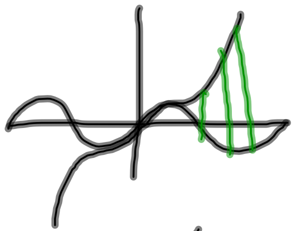
9.3 Taylor series with remainder

how many terms

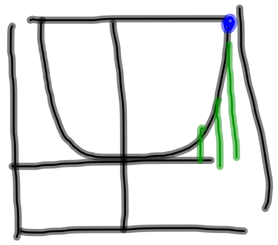
What is the fifth order Maclaurin series for f(x)=sin(x)? What is the maximum error when approximating sin(x) on $[-\pi,\pi]$

Solve graphically and numerically

$$\sin x \approx x - \frac{x^3}{3!} + \frac{x^5}{5!}$$



error function $$\epsilon = \left| \sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right) \right|$$



$x = \pi$

$\epsilon \approx .52$

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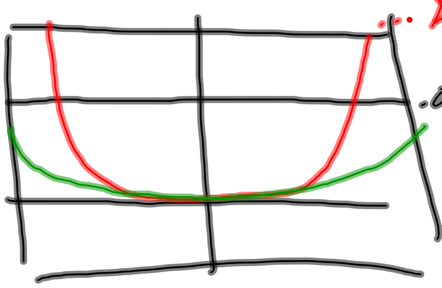
guess & check

How many terms are needed in the Maclaurin series for sin(x) in order to approximate sin(x) within .0001 on the interval $[-\pi,\pi]$

max error

$$\left| \overbrace{\sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right)}^{f_1(x)} \right| < .0001$$

$f_2(x) = .0001$



$\cdots \frac{x^{11}}{11!}$ no

.0001

yes $\cdots \frac{x^{13}}{13!}$

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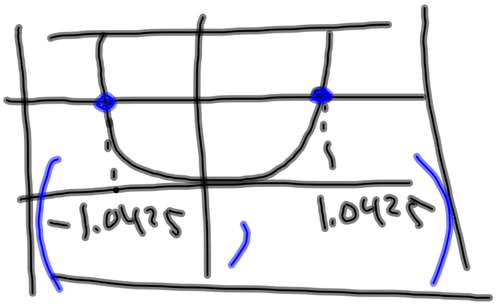
On what interval does the third order Maclaurin series approximate sin(x) within .01?

max error

$$\overset{\text{max error}}{\epsilon} = \left| \sin x - \overset{f1}{\left(x - \frac{x^3}{3!}\right)} \right| \overset{f2}{<} .01$$



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Taylor's Remainder Estimation Theorem

use n^{th} order poly.

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2} \dots \frac{f^n(a)(x-a)^n}{n!}$$

$$\boxed{\epsilon \le \left| \frac{M(x-a)^{n+1}}{(n+1)!} \right|}$$

M is the max value of $f^{n+1}(x)$ on the interval

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The approximation $\ln(1+x) \approx x - \frac{x^2}{2}$ is used when x is small.

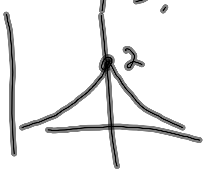
Use the Remainder Estimation Theorem to get a bound for the maximum error when $|x| \le .01$ Support the answer graphically.

$-.01 \le x \le .01$ interval

$f' = \frac{1}{1+x}$ $\quad \epsilon \le \left| M \frac{x^3}{3!} \right|$

$f'' = \frac{-1}{(1+x)^2}$ $\quad \epsilon \le \left| \frac{2x^3}{3!} \right|$

$f^3 = \left| \frac{2}{(1+x)^3} \right|$



M is max of $|f^3(x)|$ on $[-.01, .01]$

$M = 2$

max error $= \frac{2 \cdot .01^3}{3!}$

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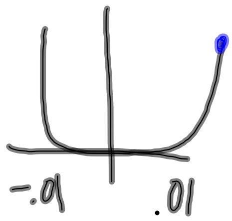

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27. $f(x) = \sqrt{1+x} = (1+x)^{\frac{1}{2}}$ $\quad f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}$ $\quad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ $\quad f''(0) = -\frac{1}{4}$

$f^3(x) = \frac{3}{8}(1+x)^{-5/2}$ $\quad f^3(0) = \frac{3}{8}$

a) $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{4}\frac{x^2}{2!} + \frac{3}{8}\frac{x^3}{3!}$

b) $\int \sqrt{1+x^2} = \int 1 + \frac{1}{2}x^2 - \frac{1}{4}\frac{x^4}{2!} + \frac{3}{8}\frac{x^6}{3!}$

c) $h'(x) = \sqrt{1+x^2}$ $\quad h(0) = 5$

$h(x) = \int \sqrt{1+x^2}\,dx = x + \frac{x^3}{2 \cdot 3} - \frac{x^5}{4 \cdot 2! \, 5} + \frac{3x^7}{8 \cdot 3! \cdot 7} + C$

$5 = C$

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9. $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\cos(x+2) = \cos 2 \cos x - \sin 2 \sin x$$

$$= \cos 2\left(1 - \frac{x^2}{2}\right) - \sin 2\left(x - \frac{x^3}{3!}\right)$$

$$= \cos 2 - \cos 2 \cdot \frac{x^2}{2} - \sin 2 \cdot x + \sin 2 \frac{x^3}{3!}$$

maclaurin series centered on 0

$$\cos(x+2) \approx \cos 2 - \sin 2 \cdot x - \cos 2 \cdot \frac{x^2}{2}$$

$$\cos(x+2) \approx 1 - \frac{(x+2)^2}{2} + \frac{(x+2)^4}{4!}$$

Taylor series centered on $x = -2$

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